A Method to Increase Class Separation in the HS Plane for Color Segmentation Applications

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Abstract – In this paper, an alternative to increase the separation between two classes (object and background) projected in the HS color plane (HSI space) is presented. Our proposal is a preprocessing stage that improves the segmentation process of color image sequence frames in real-time. The authors propose to perform a linear transformation to each frame in the RGB space that consists on adding a vector in RGB components (color vector). The optimal color vector to add maximizes the class separation in the HS plane, improving the segmentation process. The improvement in the class separation is achieved for the non-linearity of the transformation between the RGB and the HSI spaces. The $C_1 C_2$ chromatic sub-space (YC₁C₂ space) is used to obtain this optimal vector to add. Its effectiveness has been tested especially in images with a reduced contrast between the colors of the object and background and when the size of the object to segment is very small in comparison with the size of the captured scene.

<u>*Keywords*</u> – HSI and YC_1C_2 spaces, color segmentation, class separation, pixel classification.

I. INTRODUCTION

In the image processing field, the applications that perform clustering and/or object segmentation processes are usually based on a pixel-by-pixel classification. This classification is often carried out by means of a metric distance, such as the *Mahalanobis distance* [1], and/or its use in discriminant functions based on Gaussian *pdfs* [2].

Other works use criteria related to the *scatter matrix* to perform the pixel-to-pixel classification [3], [4]. In any case, the calculation or estimation of the covariance matrixes of the involved classes plays a fundamental role in the operation and performance of the classification, because they determine the class reliability. In the last works [3], [4] a selection of the components in the feature space (hybrid color space) is performed in order to increase the class separation, and therefore, improve the classification.

In this paper, the authors propose to increase the class separation by means of an object/background pre-processing, thus enhancing the contrast in the HS plane between the colors corresponding to the object to segment and the scene background. This is carried out by properly coloring the image with a color vector in the RGB space.

On the other hand, to avoid the variances calculation, a modeling to estimate the Hue (H) and Saturation (S) deviations is used in this work. This estimation is performed in a reduced processing time, because it is designed to be applied in real time.

The mathematical relationships between the HSI [5] and the YC_1C_2 [6-8] color spaces are used to obtain the color vector to add and to model the dispersions. An estimation of the hue and saturation deviation is performed in [9] for the Smith's HSI transformation. In [11], the estimations of the component deviations of several color spaces from the values of the R, G and B and its respective deviations are obtained.

This paper has been organized as follows: section II describes the method proposed to increment the separation between classes. Section III presents the experimental results and section IV contains the conclusions and future works.

II. PROPOSED METHOD

The objective of the proposal presented in this work is to increase the separation between the two classes projected in the HS plane, by adding an optimal vector of RGB components. This optimal vector to add is obtained for each captured image in the RGB space and produces the maximum separation between the classes to segment in the plane where the segmentation must be performed: HS plane.

A. Algorithm Overview

If the original image is denoted by I, the optimal color vector to add by \mathbf{i}_{r} , and the colored image resulting of the addition, by \mathbf{I}_{i} , the following expression is fulfilled:

$$\mathbf{I}_{i} = \mathbf{I} + \mathbf{i}_{\mathbf{r}} \tag{1}$$



Fig. 1. General block-diagram of the proposed algorithm to calculate the optimal color vector.

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The determination of the vector \mathbf{i}_r for each captured *RGB* image is done following different steps, as shown in Fig. 1. These steps are:

1) From the captured RGB image (I), significant samples (seeds) are obtained from both the object to segment (class O) and the background (class B). Our starting point is that the classes are already identified by some method of classification, as k-means algorithm. We will refer to the object class in the RGB space as $O_{RGB}=\{\mathbf{r}_{Ok}\}$; k=1,2,..N, while $B_{RGB}=\{\mathbf{r}_{Bk}\}$ will refer to the background class, where N is the number of pixel seeds randomly taken for the classes.

2) For each sample, the transformation from the RGB space to the YC_1C_2 space is done. After this transformation, the resulting classes will be referred to as $O_{CIC2} = \{c_{Ok}\}$ and $B_{CIC2} = \{c_{Bk}\}$ for k=1,2,..N.

3) Making use of the properties and relationship between the *HSI* and YC_1C_2 color spaces, the optimal location of the classes in the C_1C_2 plane is obtained by finding the optimal location of their respective mean vectors, i.e., \mathbf{c}_{i0opt} and \mathbf{c}_{iBopt} .

4) From the $\mathbf{c_{iOopt}}$ and $\mathbf{c_{iBopt}}$ vectors, its corresponding ones in the *RGB* space (denoted by $\mathbf{r_{iO}}$ and $\mathbf{r_{iB}}$) can be obtained. The optimal color vector to add can be calculated using these vectors and the mean ones ($\mathbf{r_O}$ and $\mathbf{r_B}$) of the original classes (O_{RGB} , B_{RGB}), with any of the following expressions:

$$\mathbf{i}_{\mathbf{r}} = \mathbf{r}_{\mathbf{i}\mathbf{O}} - \mathbf{r}_{\mathbf{O}} , \ \mathbf{i}_{\mathbf{r}} = \mathbf{r}_{\mathbf{i}\mathbf{B}} - \mathbf{r}_{\mathbf{B}}$$
(2)

5) Once the optimal color vector has been obtained, the new colored image, I_i , can be calculated using (1). Finally, I_i is transformed from the *RGB* space to the *HS* sub-space, where the segmentation is done. The color vector i_r has its effects in the *HSI* space due to the nonlinearity of this space.

B. Class Separation Measurement

The Fisher Ratio (FR) [3], [4], [12] is frequently used to measure the efficiency of the class separation in classification systems. This ratio simultaneously quantifies the inter-class and intra-class scatter. For a two-class system, it is interesting to achieve a large metric distance between the class means and a minimum dispersion within each class (leading to a high FR). In this work, the FR is used as a measurement index of the effectiveness of the separation between classes for pixel classification in the HS plane. In the case of a multiclass system, the generalized FR [13] is expressed by: FR=trace $(M_w^{-1}M_b)$, where M_b refers to the inter-class (*between classes*) covariance matrix and M_w refers to the internal (*within class*) dispersion matrix of the class. Due to the circular form of the trajectory of the *H* component, this last equation cannot be directly applied. For this reason, and supposing that the correlation between *H* and *S* is small, a *FR* is calculated individually for each component [13]. As our case is bi-dimensional, it is given that:

$$FR = FR_S + FR_H \tag{3}$$

where $FR_H = \theta_h^2 / (\sigma_{HO}^2 + \sigma_{HB}^2)$ and $FR_S = d_S^2 / (\sigma_{SO}^2 + \sigma_{SB}^2)$ are the Fisher Ratios of the *H* and *S* components, respectively, being $d_S = \mu_{SO} - \mu_{SB}$ the distance between the saturation means of both classes, θ_h is the separation angle between the hue means (μ_{HO} and μ_{HB}), σ_{SO} , σ_{SB} , σ_{HO} and σ_{HB} are the standard deviations of the saturation and hue for both classes.

C. Relationships between the RGB Space, and the HS and C_1C_2 Planes

Given a vector $\mathbf{r} = [R \ G \ B]^T$ located in the *RGB* space, the *H* and *S* components of a vector \mathbf{h} [12], are given by:

$$S = (1 - 3\min(R, G, B) / (R + G + B))$$
(4)

$$H = \begin{cases} \gamma, B \le G \\ 2\pi - \gamma, B > G \end{cases}; \ \gamma = \cos^{-1} \left(\frac{R - \frac{1}{2G} - \frac{1}{2B}}{(R^2 + G^2 + B^2 - RG - GB - BR)^{\frac{1}{2}}} \right) (5)$$

The components in the YC_1C_2 space [6-8] from vector **r**, are:

$$\begin{bmatrix} Y \\ C_1 \\ C_2 \end{bmatrix} = \mathbf{Q} \begin{bmatrix} R \\ G \\ B \end{bmatrix}; \ \mathbf{Q} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}$$
(6)

being **Q** the transformation matrix between spaces. From (6), the components C_t and C_{ϱ} of a vector **c** are given by:

$$C_1 = R - \frac{1}{2}G - \frac{1}{2}B, \ C_2 = \sqrt{3}/2G - \sqrt{3}/2B$$
 (7)

From (7), the Chroma component, C, and the angle, H of the vector **c** in the C_1C_2 plane can be calculated using:

$$C = (C_1^2 + C_2^2)^{1/2} = (R^2 + G^2 + B^2 - RG - GB - BR)^{1/2}$$
(8)

$$H = \begin{cases} \alpha, B \le G \\ 2\pi - \alpha, B > G \end{cases}; \ \alpha = \cos^{-1} \left(\frac{R - \frac{1}{2G} - \frac{1}{2B}}{(R^2 + G^2 + B^2 - RG - GB - BR)^{\frac{1}{2}}} \right)$$
(9)

Therefore, relating (5) and (9), it can be observed that a vector in the RGB space can be projected in the HS and C_1C_2 planes with the same angle but a different module, that is: H=H and $S\neq C$. It can also be demonstrated that the relationship between S and C is:

$$S = \frac{2Cf(H)}{3I}; \ f(H) = \begin{cases} \cos(H - \pi/3); \ (0 < H \le 2\pi/3) \\ \cos(H - \pi); \ (2\pi/3 < H \le 4\pi/3) \\ \cos(H - 5\pi/3); \ (4\pi/3 < H \le 2\pi) \end{cases}$$
(10)

where *I* is the intensity from *HSI*, which coincides with the *Y* component, f(H) is a weighting function that depends on the *H* component and $f(H) \in [\frac{1}{2}, 1]$. This f(H) function generates a three lobe curve in the *HS* plane (Fig. 2), delimitated by the discontinuities corresponding to the three color sectors of the plane: $0-2\pi/3$, $2\pi/3-4\pi/3$ and $4\pi/3-2\pi$.

After all the above discussion, given two mean vectors in the RGB space, \mathbf{r}_0 and \mathbf{r}_B , the resulting projection vectors in the C_1C_2 (\mathbf{c}_0 and \mathbf{c}_B) and HS planes (\mathbf{h}_0 and \mathbf{h}_B) fulfill:

$$\theta_c = \theta_h = \theta, \ \left\| \mathbf{c}_{\mathbf{O}} \right\| \neq \left\| \mathbf{h}_{\mathbf{O}} \right\|, \ \left\| \mathbf{c}_{\mathbf{B}} \right\| \neq \left\| \mathbf{h}_{\mathbf{B}} \right\|$$
(11)

$$\left\|\mathbf{d}_{\mathbf{c}}\right\|^{2} = g_{1}\left(\mathbf{c}_{\mathbf{O}}, \mathbf{c}_{\mathbf{B}}, \theta_{c}\right) = \left\|\mathbf{c}_{\mathbf{O}}\right\|^{2} + \left\|\mathbf{c}_{\mathbf{B}}\right\|^{2} - 2\left\|\mathbf{c}_{\mathbf{O}}\right\|\left\|\mathbf{c}_{\mathbf{B}}\right\|\cos\theta_{c} \quad (12)$$

$$\left\|\mathbf{d}_{\mathbf{h}}\right\|^{2} = g_{2}\left(\mathbf{c}_{\mathbf{O}}, \mathbf{c}_{\mathbf{B}}, \theta_{h}, \mu_{IO}, \mu_{IB}, f(H)\right)$$
(13)

where θ_c is the angle formed by the vectors $\mathbf{c}_{\mathbf{O}}$ and $\mathbf{c}_{\mathbf{B}}$, θ_h is the one formed by $\mathbf{h}_{\mathbf{O}}$ and $\mathbf{h}_{\mathbf{B}}$. $\mathbf{d}_{\mathbf{c}}$ and $\mathbf{d}_{\mathbf{h}}$ are the distance vectors between $\mathbf{c}_{\mathbf{O}}$ and $\mathbf{c}_{\mathbf{B}}$, and $\mathbf{h}_{\mathbf{O}}$ and $\mathbf{h}_{\mathbf{B}}$, respectively. μ_{IO} and μ_{IB} are the intensity means of both classes that correspond to the $\mathbf{h}_{\mathbf{O}}$ and $\mathbf{h}_{\mathbf{B}}$ vectors.

On the other hand, it is important to note that when adding a vector $\mathbf{i}_{\mathbf{r}}$ (color vector to add) to the vectors \mathbf{r}_{0} and \mathbf{r}_{B} in the RGB space, the distance vector $\mathbf{d}_{\mathbf{c}}=\mathbf{c}_{0}-\mathbf{c}_{B}$ in the $C_{1}C_{2}$ (linear) plane remains constant. Therefore, adding $\mathbf{i}_{\mathbf{r}}$ in the RGBspace results in a translation of the classes in the $C_{1}C_{2}$ plane. This effect can be achieved with a translation vector $\mathbf{i}_{\mathbf{c}}$ (corresponding to $\mathbf{i}_{\mathbf{r}}$) directly added in the $C_{1}C_{2}$ plane.

Summarizing: to obtain the value of the color vector to be added in the RBG space, and, therefore, a particular



Fig. 2. Correspondence between the mean vectors of the C_1C_2 plane and the HS plane. The vector $\mathbf{d}_{\mathbf{c}}$ before and after the addition of a color vector.

separation between the classes in the HS plane, the authors suggest the use of the relationship between the **h** vector components in the HS plane and their corresponding **c** vector components in the C_1C_2 plane, given by (9) and (10), and the relationship between pairs of vectors in these planes, given by (11), (12) and (13).

D. Increase of the Distance between the Class Means

In order to increase the distance between the class means, the authors propose the relocation of the vectors \mathbf{c}_0 and \mathbf{c}_B in the $C_1 C_2$ plane by means of the addition of the color vector \mathbf{i}_r . This increase is possible due to properties of the difference vector (\mathbf{d}_c) between both vectors: the constant values of the magnitude ($||\mathbf{d}_c||$) and orientation (ϕ) (*mean invariants*). Thus, in Fig. 2, an example of the correspondence between the vectors \mathbf{c}_0 and \mathbf{c}_B in the $C_1 C_2$ plane and the vectors \mathbf{h}_0 and \mathbf{h}_B in the HS plane is shown.

These relationships after the addition of a vector \mathbf{i}_r (relocated vectors \mathbf{c}_{iO} , \mathbf{c}_{iB} , and \mathbf{h}_{iO} , \mathbf{h}_{iB}), and the vector \mathbf{d}_c before and after the translation due to the addition of the vector \mathbf{i}_r are also shown. The invariance in magnitude and phase of \mathbf{d}_c can be observed here.

From now on, the "i" or "*i*" subscript indicates that the vector $\mathbf{i}_{\mathbf{r}}$ has been added. Fig. 2 depicts how the translation of the vector $\mathbf{d}_{\mathbf{c}}$ has favored the separation of the mean vectors of the classes in both components (*H* and *S*), because $\theta_i > \theta$, and $(||\mathbf{h}_{i0}|| - ||\mathbf{h}_{iB}||) > (||\mathbf{h}_0|| - ||\mathbf{h}_B||)$. As can be observed, the separation angle between the vectors after the addition of the color vector has increased $(\theta_i > \theta)$, however, the vector modules (saturation) decrease $(||\mathbf{h}_0|| > ||\mathbf{h}_{i0}||, ||\mathbf{h}_B|| > ||\mathbf{h}_{iB}||)$, since there is an unavoidable compensation effect given by (12).

a) Increase in the Separation between the Hue Means

The possibility of utilizing d_h to obtain the separation between the hue means is rejected due to the discontinuities presented by $\mathbf{d}_{\mathbf{h}}$. On the contrary, the distance function $\|\mathbf{d}_{\mathbf{c}}\|$ defined by \mathbf{c}_{i0} and \mathbf{c}_{iB} (12) does not present discontinuities. Therefore, the proposed algorithm has been parameterized as a function of the separation angle (θ_i) between these vectors, $\mathbf{c_{iO}}$ and $\mathbf{c_{iB}}$. In our case, the optimal angle θ_i is obtained from an observation function that measures the effectiveness of the separation between the classes in different locations in the HSplane. This function will be described in the section F. When the angle of separation (θ_i) is maximum, it coincides with the angle whose bisector is a straight line p, that passes through the origin of coordinates and is perpendicular to the straight line, m, whose director vector is d_c (Fig. 3). This implies that the modules of both vectors \mathbf{c}_{iO} and \mathbf{c}_{iB} are equal, i.e., $(||\mathbf{c}_{i0}|| = ||\mathbf{c}_{iB}|| = C_i)$ (forced location).

b) Increase in the Separation between the Saturation Means

Given two vectors in the *HS* plane, for example \mathbf{h}_{i0} and \mathbf{h}_{iB} , and taking into account the restriction imposed in the previous paragraph, (forced location), we analyze how the



Fig. 3. Location of \mathbf{c}_{i0} and \mathbf{c}_{iB} vectors in the $C_t C_2$ plane, once the color vector has been added causing a forced location.

value of the saturation difference, d_{S} , varies once $\mathbf{i}_{\mathbf{r}}$ is added, $d_{Si} = \mu_{SiO} - \mu_{SiB} = ||\mathbf{h}_{i0}|| - ||\mathbf{h}_{iB}||$, as θ_i changes. In our case, as $||\mathbf{c}_{i0}|| = ||\mathbf{c}_{iB}|| = C_i$, according to (10), an expression of d_{Si} depends mainly on f(H) and on the mean intensities of both classes (μ_{IiO} , μ_{IiB}). As the HS plane has three color sectors, 9 possible combinations of d_{Si} are generated. For example, in the case of the Fig. 3, \mathbf{h}_{iO} is located in lobe 1 (overlapped with \mathbf{c}_{iO}) and \mathbf{h}_{iB} in lobe 2 (overlapped with \mathbf{c}_{iB}), so, from (10), the following expression can be demonstrated:

$$d_{Si} \simeq (\mu_{C1iO} + \sqrt{3}\mu_{C2iO})/3\mu_{IiO} + \mu_{C1iB}/3\mu_{IiB}$$
(14)

where, μ_{Clio} and μ_{C2io} are the C_1 and C_2 components of the vector \mathbf{c}_{i0} , and μ_{CliB} and μ_{C2iB} are ones of the vector \mathbf{c}_{iB} .

E. Reliability of the Class Dispersions

The analysis of the class reliability is of great importance in the class separation process and it is necessary in order to obtain the class separation measurement function in this paper. Therefore, we must consider how the dispersions of saturation and hue of the classes in the HS plane are affected as they are translated in the C_1C_2 plane, as a result of the addition of the color vector $\mathbf{i}_{\mathbf{r}}$. We obtain the value of the deviation of the hue and saturation of a class as a function of its mean (μ_{C1} and μ_{C2}) by means of a modeling.

a) Modeling of the Hue Dispersion

Given that the angle of a vector **c** in the $C_1 C_2$ plane and the one of a vector \mathbf{h} in the HS plane is the same (overlapped vectors), the variation of the angular dispersion in the $C_1 C_2$ plane corresponds with the variation of the hue dispersion in the HS plane. The uncertainty ellipse of the class in the C_1C_2 plane is obtained by means of a geometric-analytical formulation and, then, the angular deviation or hue deviation (σ_{H}) is approximated by half of the angle between the two straight lines tangent to the uncertainty ellipse (one on each side of the angular mean, $\mu_{\rm H}$) that also pass through the origin of the plane. For a class characterized by its statistical data: $\mathbf{c} = [\mu_{C1} \ \mu_{C2}]^T$ as mean vector, and covariance matrix, $\boldsymbol{\Sigma}$, the parameters of the class uncertainty ellipse, called invariants of hue dispersion, are: $\omega = \tan^{-1}(C_{2u}/C_{1u})$, $u = \sqrt{\lambda_u}$ and $l = \sqrt{\lambda_l}$. Here, ω is the angle of the ellipse major axis with respect to the horizontal axis (C_l axis), u and l are

the semimajor and semiminor axes of the ellipse, C_{1u} and C_{2u} are the components of the eigenvector corresponding to the highest eigenvalue (λ_u) of Σ , and λ_l is the lowest one.

The estimation of σ_H as a function of the parameters μ_{Cl} and μ_{C2} starts by the choosing one of the tangency points of the two tangent lines with the ellipse. Four tangency points can be obtained relating the partial derivative of the ellipse equation in the $C_1 C_2$ plane with the tangent lines, knowing that these pass through the origin of the plane. The choice of a particular point depends on the location of the ellipse within the plane, because all of them may be valid. If H_{tn} ; n=1, 2, 3,4, are the angles produced by these tangency points, the hue deviation is approximated by:

$$\sigma_{H} \simeq \frac{1}{2} \left(\max(\vec{d}(\mu_{H}, H_{tn})) - \min(\vec{d}(\mu_{H}, H_{tn})) \right); n = 1, 2, 3, 4 \quad (15)$$

where $d(\mu_H, H_{tn})$ is the *directed distance*, created to avoid the problems of the cyclic property of the Hue seen in [6].

b) Modeling of the Saturation Dispersion

The saturation dispersion is affected in minor degree by the class translations in the C_1C_2 plane, than for the variation of the class sample intensities (I=Y). The reason is that it can be demonstrated that saturation is a linear function of the C_1 and C_2 components, besides, (10) shows that varies inversely with I. An equation to model the behavior of the saturation deviation (σ_S) of a class when it moves in the HS plane, as a function of μ_{C1} and μ_{C2} , is possible to obtain using (10). Knowing that C_{1k} and C_{2k} components can be expressed as a function of its respective class means, i.e., $C_{1k}=(\mu_{C1}+\Delta_k)$ and $C_{2k}=(\mu_{C2}+\nabla_k)$; k=1,2,..N, the saturation variance is obtained:

$$\sigma_{S}^{2} \simeq k_{N} \left(a_{s} \mu_{C1}^{2} + d_{s} \mu_{C1} + b_{s} \mu_{C1} \mu_{C2} + e_{s} \mu_{C2} + c_{s} \mu_{C2}^{2} + f_{s} \right)$$
(16)

where $k_N = f(N)$ and $\{a_s, b_s, c_s, d_s, e_s, f_s\} = f(\Delta, \nabla, \mathbf{Y})$ depending on the color sector considered. Here, $\Delta = [\Delta_1 \ \Delta_2 \ \dots \ \Delta_N]^T$, $\nabla = [\nabla_1 \ \nabla_2 \ \dots \ \nabla_N]^T$, and \mathbf{Y} is a diagonal matrix formed by Y_1 , $Y_2, \dots \ Y_N$. The parameters Δ , ∇ and \mathbf{Y} remain constant in the translation and are called *invariants of saturation dispersion*. As can be seen, Δ and ∇ indirectly represent the mean deviations of the class components in the $C_1 C_2$ plane, and $\mu_Y = \mu_I = \text{tr}(\mathbf{Y})/N$.

F. Algorithm to Obtain the Optimal Vector (i_r) to Add in the RGB Space

This section presents the strategy used to obtain, in C_1C_2 plane, the mean vectors that maximize the separation between the classes in the *HS* plane ($\mathbf{c}_{i\mathbf{0}opt}$ and $\mathbf{c}_{i\mathbf{B}opt}$). This section constitutes the main stage in the Fig.1 diagram. The proposal to obtain these optimal vectors, $\mathbf{c}_{i\mathbf{0}opt}$ and $\mathbf{c}_{i\mathbf{0}opt}$ consists of different steps. It includes an iterative algorithm to obtain a set of locations for $\mathbf{c}_{i\mathbf{0}}$ and $\mathbf{c}_{i\mathbf{B}}$ in C_1C_2 plane. The location of each vector will be parameterized by the angle θ_i as explained in section *D*. The optimal value of θ_i , θ_{opt} , is obtained from the set θ_{in} (θ_{i1} , θ_{i2} ...) associated to the set of indexes of class separation measurement β_{HSn} (β_{HS1} , β_{HS2} ...), step *e*. The process begins obtaining the mean vectors of each class in $C_1 C_2$ plane, i.e., $\mathbf{c_0} = \mathbf{E} \{ \mathbf{c_{0k}} \}$ and $\mathbf{c_B} = \mathbf{E} \{ \mathbf{c_{Bk}} \}$; k=1,2,...N. The called *mean invariants* of vector $\mathbf{d_c}$ are obtained from $\mathbf{c_0} = [\mu_{C10} \ \mu_{C20}]^T$ and $\mathbf{c_B} = [\mu_{C1B} \ \mu_{C2B}]^T$ using $||\mathbf{d_c}|| = (d_{C1}^2 + d_{C2}^2)^{1/2}$ and $\phi = \cos^{-1}(d_{CI}/||\mathbf{d_c}||)$, where $[d_{CI} \ d_{C2}]^T = \mathbf{c_0} - \mathbf{c_B}$.

The iterative process consists of the following 5 steps:

a) Forced location of the mean vectors in the C_1C_2 plane

The original vectors $\mathbf{c}_{\mathbf{0}}$ and $\mathbf{c}_{\mathbf{B}}$ are relocated (forced) in the $C_1 C_2$ plane using the invariants ($||\mathbf{d}_{\mathbf{c}}||, \phi$), knowing that:

$$C_i = \left\| \mathbf{c}_{i\mathbf{O}} \right\| = \left\| \mathbf{c}_{i\mathbf{B}} \right\| = \left\| \mathbf{d}_{\mathbf{c}} \right\| / 2\sin(\theta_i / 2).$$
(17)

The Cartesian components of these vectors (see Fig. 3), particularized for the vector c_{io} , are given by:

$$\mu_{C1iO} = C_i \cos(\mu_{HiO}), \ \mu_{C2iO} = C_i \sin(\mu_{HiO})$$
(18)

where μ_{HiO} is the angle of the vector that can be expressed by:

$$\mu_{HiO} = \pi/2 + \phi - \theta_i/2. \tag{19}$$

The iterative algorithm is initialized with $\theta_i = \theta$. In each iteration (*j*) the value of θ_i , is increased: $\theta_i(j) = \theta_i(j-1) + \Delta \theta$.

b) Verification of the c_{i0} and c_{iB} vectors locations validity

For each increase of θ_i , the validity of the locations of the vectors $\mathbf{c_{i0}}$ and $\mathbf{c_{iB}}$ is verified by checking if the values of the components of the corresponding vectors $(\mathbf{r_{i0}}, \mathbf{r_{iB}})$ in *RGB* space are lower than 1. If the locations are valid, the value of θ_i will be included in the set θ_{in} .

c) Calculation of the class translation vector in the C_1C_2 plane

The translation vector \mathbf{i}_{c} is obtained for each value of θ_{in} . This \mathbf{i}_{c} is responsible of the class translations from its original position to the forced location defined by θ_{in} and is given by: $\mathbf{i}_{c}=\mathbf{c}_{i0}-\mathbf{c}_{0}$. The translation of both classes in the $C_{i}C_{2}$ plane is made with \mathbf{i}_{c} . For the O class, $O_{iCIC2}=\{\mathbf{c}_{0k}+\mathbf{i}_{c}\}$; k=1,2,...N.

d) Class transformation from the C_1C_2 plane to the HS

The classes in the *HS* plane (O_{iHS} and B_{iHS}) are obtained from the translated classes O_{iCIC2} and B_{iCIC2} , using (9), (10) and knowing that I = Y.

e) The observation function: calculation of the class separation measurement index (β_{HSn}) in the HS plane

As an observation function of the separation between the classes, a normalized index of measurement (β_{HS}) has been defined from the FR described in (3), such as: $\beta_{HSn} = k_h \beta_{Hn} + (1-k_h) \beta_{Sn}$, where $\beta_{Hn} = (\sqrt{FR_H} - 1)/\sqrt{FR_H}$ and $\beta_{Sn} = (\sqrt{FR_S} - 1)/\sqrt{FR_S}$ are the measurement indexes in H and S. The proposal described in section E is used to obtain the deviations ($\sigma_{SO}, \sigma_{SB}, \sigma_{HO}$ and σ_{HB}) involved in the calculation of β_{Hn} and β_{Sn} . In order to reduce the processing times when estimating the deviations, only the expression depending on translations are used. $k_h \in [0, 1]$ is a weighting factor. Usually, H has a greater discriminating power than S, therefore $k_h > \frac{1}{2}$. This iterative process is repeated until the first non valid value of θ_{in} is generated, the pairs (β_{HSn} , θ_{in}) are registered and the θ_{opt} is obtained when β_{HSn} is maximum.

After the iterative process, the vector $\mathbf{c}_{i\mathbf{0}opt}$ or $\mathbf{c}_{i\mathbf{B}opt}$ are obtained using the θ_{opt} , by means of (17), (18) and (19), in this case $\mathbf{c}_{i\mathbf{0}opt} = [\mu_{ClOopt} \ \mu_{C2Oopt}]^T$. Its corresponding vector in the RGB space is: $\mathbf{r}_{i\mathbf{0}} = \mathbf{Q}^{-1} [\mu_{YiO} \ \mu_{ClOopt} \ \mu_{C2Oopt}]^T$ where, μ_{YiO} is the intensity mean of the class O already translated in the C_1C_2 plane. The \mathbf{i}_r vector is obtained with this $\mathbf{r}_{i\mathbf{0}}$ applying (2). In our case $\mu_{YiO} = \mu_{YO} (\mu_{IiO} = \mu_{IO})$, because we want that the intensity mean of the original image (I) and the colored one (I_i) are equal, implying that $E\{\mathbf{i}_r\}=0$.

III. EXPERIMENTAL RESULTS

This method to increase the class separation has been applied to several real images to validate our proposal. The effect produced in the new image, I_i , when we add the color vector i_r to the original image is a higher concentration of the pixel colors around the mean color of each class. That is, the addition of the vector i_r equalizes the histogram of the captured image, both in *H* and *S*, see Fig. 4. This equalization has a concentration effect in each class, increasing the distance between their centers. The increment in the class separation can be directly observed locating the classes in the histogram before and after the addition of the color vector. Fig. 5 depicts the histograms corresponding to the classes before (a) and after (b) the addition of the vector i_r . A remarkable increment in the separation of the hue components of both classes can be observed.



Fig. 4. 2D Histograms, a) image I, b) image I_i.

The experimental results have been quantified by means of the FR defined in (3). Table 1 shows the values of FR for 5 cases of the bank of images used in the experimental tests.

In the tests made, the following data have been used: $k_h=0.87$, $\Delta\theta=1^{\circ}$ and N=50, 100, 200, 500. The tests have been carried out in a PC with an Intel Centrino processor at 1.5GHz. The processing time obtained for 48 iterations (average of the 5 cases) when calculating the variances (CPT)



Fig. 5. Histogram of the object and background classes: a) before the addition of the color vector, b) after the addition of the color vector.

is higher (60ms, 92ms, 175ms and 554ms) than the processing time if the variances are estimated (EPT) by means of the modeling proposed (20ms approximately). The EPT is notably minor and practically constant for the different values of N.

Table 1. FR results for 5 cases of the image bank

Case	FR	FR (color vector added)	% increase
1	49.15	112.32	128.53
2	74.67	246.82	230.52
3	11.18	21.84	95.31
4	68.08	1826.02	2581.97
5	100.82	214.46	112.71

Finally, Fig. 6 depicts an example of segmentation of an image sequence after applying our proposal with the objective of improving the segmentation. The skin color of a person generating sign language is segmented. The *Mahalanobis distance* is used in the segmentation process to classify the pixels according to the smaller distance to the object or background class. The figure shows: a) original image, b) segmentation without applying the method, c) segmentation applying the proposed method, where a remarkable improvement in the segmentation accuracy can be observed.



Fig. 6. Segmentation results of an image sequence of a person generating the Spanish Sign Language.

IV. CONCLUSION

A method to increase the separation between two classes in the HS plane has been proposed. The experimental results obtained show that the addition of an optimal color vector to the image guarantees good results in the class separation, and improves the segmentation of the desired object. Its practical implementation is simple and effective in real-time applications, because it has a low computational cost for the H and S dispersion modeling. Currently our research is focused on the addition of a color vector with non-zero mean, and on applying higher order transformations that imply scales and rotations of the classes.

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